THE MONTE CARLO METHOD

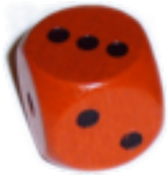
John von Neumann (1903 - 1957), mathematician. Born in Budapest into a wealthy family his genius was recognised early. At the age of six he was fluent in Hungarian, German, French, Latin and Greek. He studied chemistry and mathematics in Hungary, Germany and Switzerland and still in his twenties started lecturing in Berlin, Hamburg and Prince-ton.
In 1933 he moved, like Albert Einstein, to Princeton, where he became the mathematical shooting star of his era and achieved celebrity status. He died of cancer at the age of 54, two years after Einstein.
He is regarded as the most important and broadest mathematician of the first half of the 20th century. He developed the mathematics of quantum theory, the basics of game theory, contributed to the development of computers and the theory of simulation. During World War II he was consultant of the armed forces and was involved in the construction of the atomic bomb in Los Alamos.

The outcomes of most of the processes in the real world are not completely predictable, they are partly controlled by chance. Nevertheless it might be important to know with what likelihood a certain outcome can be expected. But real processes are usually too complicated to be modelled and calculated, so they have to be simulated using random number generators. If a simulation is repeated 1,000 times and a certain outcome A occurs 200 times the probability p(A) is estimated to be close to $\frac{200}{1000} = \frac{1}{5}$.
This method was used in Los Alamos (New Mexico, USA) during the "Manhattan Project" which had been set up by the American government at the start of World War II to develop the atomic bomb. It was the only way to predict the extremely complicated and dangerous sequence of events when splitting atoms.
For this kind of simulation random numbers are needed. They can be produced either with tables (using the sequence of the decimal places of an irrational number, i.e. π) or with a random number generator as it is implemented in a modern calculator.

Ti-nspire for example:

randInt(a,b)	produces random integers $\in [a,b]$	(a, b integers ≥ 0)
rand()	produces random numbers $\in [0,1]$	

Random numbers produced by a machine are not random in the deepest sense of the word. The machine will produce exactly the same "random" numbers again if it is in exactly the same state, i.e. if the storage space is filled identically. But for our purposes the numbers produced are "random" enough.



1 Simulate 40 throws with a red and a white die.

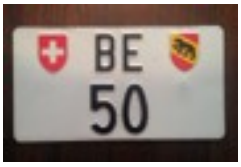
a) What is the probability that the sum of the pips is 2?

b) What is the probability that the sum of the pips is 5?

2 Calculate theoretically the probabilities of 1 .

3 Each Sunday morning the same five people come to the newsagent's at the railway station to fetch the newspaper. They arrive independently at any time between 9.00 and 9.59 and stay exactly 1 minute to have a chat with the newsagent.
What is the probability that at least two of these five customers meet?
Solve with the Monte Carlo method and simulate 20 Sundays.

4 Assuming there are exactly 99,999 cars in a canton with number plates from 1 up to 99,999.
What are the chances that a randomly chosen number plate shows at least one 5?
Solve with the Monte Carlo method and simulate 20 number plates.



5 Solve 4 theoretically.

6 On a mild Saturday morning at a lake five hunters shoot at the same time at a flight of five ducks. Each hunter chooses his prey at random and independent from the others, and each hunter is a perfect rifleman who hits the target under all circumstances.
What are the chances that exactly two ducks are killed on such a Saturday morning?
Solve with the Monte Carlo method and simulate 20 Sundays.